Statistics

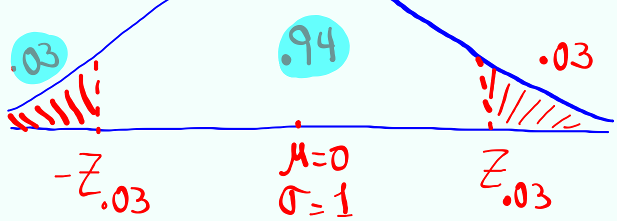
Lecture 12

Feb 19-8:47 AM

Find $z_{\alpha/2}$ for 94% C-level.

Middle Area .94

each tail $\frac{1-.94}{2} = .03$



$z_{.03} = \text{invNorm}(.97, 0, 1) \approx \boxed{1.881}$

Jan 30-4:35 PM

In a Sample of 280 Callers, $\boxed{85\% \text{ of them}}$ were happy with service they got.

$n = 280$ $\quad x = n\hat{p} = 280(.85)$ $\boxed{x = 238}$

$\hat{p} = .85$ $\quad$ if decimal → Round-up

Find 98% Conf. interval for the prop. of all Callers that are happy with Service.

C-level: .98

1 - PropZInt

$x = 238$

$n = 280$

C-level: .98

$\boxed{.80 < P < .90}$

$E = \dfrac{.90 - .80}{2}$

$\boxed{E = .05}$

$\hat{p} = \dfrac{.90 + .80}{2}$

$\boxed{\hat{p} = .85}$

we are 98% Confident that between 80% & 90% of all Callers are happy with Service.

Jan 30-4:38 PM

How to determine minimum Sample Size needed:

$$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$$ Find n

with Some algebra

$$\boxed{n = \hat{p}\hat{q}\left(\frac{Z_{\alpha/2}}{E}\right)^2}$$

Always round-up when decimal

If $\hat{p}$ & $\hat{q}$ are both unknown, we use .5 for each,

$$\boxed{n = .25\left(\frac{Z_{\alpha/2}}{E}\right)^2}$$

Always round-up

Jan 30-4:46 PM

Find minimum Sample Size needed for number of Callers if we wish to construct 95% Conf. interval with 4% margin of error.

$\hat{p} = .85$

$\hat{q} = .15$

$E = .04$

New C-level: .95

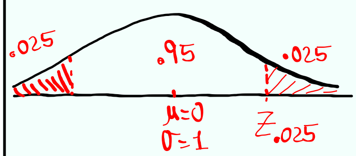
$n = \hat{p}\hat{q}\left(\frac{Z_{\alpha/2}}{E}\right)^2$

$= (.85)(.15)\left(\frac{1.960}{.04}\right)^2$

$= 306.1275$

Round up

$\boxed{n = 307}$



$Z_{.025} = \text{invNorm}(.975, 0, 1) = \boxed{1.960}$

Suppose $\hat{p}$ & $\hat{q}$ were both unknow

$n = .25\left(\frac{Z_{\alpha/2}}{E}\right)^2 = .25\left(\frac{1.960}{.04}\right)^2 = 600.25$

Round-up

$\boxed{n = 601}$

Jan 30-4:51 PM

In a Sample of 150 items checked, 12.5% of them were below the standard set by the Company.

$n = 150$

$\hat{p} = .125$

$\Rightarrow x = n\hat{p} = 150(.125)$ $\boxed{x = 19}$

if decimal → Round-up

Find Conf. interval for the prop. of all items that are below Standards set by the Company.

NO C-level

⇒ Use .95

1-PropZInt

$x = 19$

$n = 150$

C-level: .95

$\boxed{.07 < p < .18}$

we are 95% Confident that between 7% to 18% of all items are below company standards.

$E = \frac{.18 - .07}{2} = \boxed{.055}$

$\hat{p} = \frac{.18 + .07}{2} = .125$

Jan 30-5:00 PM

Find minimum Sample Size needed if we wish to be 90% Confident and margin of error to be within 8%.

$\hat{p} = .125$

$\hat{q} = .875$

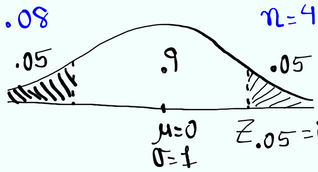
C-level $= .9$

$E = .08$

$n = \hat{p}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2$

$= (.125)(.875)\left(\frac{1.645}{.08}\right)^2$

$n = 46.245$ $\boxed{n = 47}$



$z_{.05} = \text{invNorm}(.95, 0, 1)$

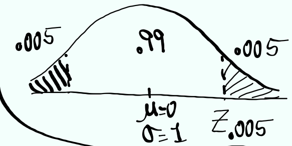
$= \boxed{1.645}$

Let's redo with C-level .99, E = 4%.

$n = (.125)(.875)\left(\frac{2.576}{.04}\right)^2$

$n = 453.6175$

$\boxed{n = 454}$



$z_{.005} = \text{invNorm}(.995, 0, 1)$

Jan 30-5:08 PM

Confidence Interval for $\underbrace{\text{Population Mean}}_{\mu}$

$$\bar{x} - E < \mu < \bar{x} + E$$

Sample Mean Point-Estimate

Margin of error

Case I: σ Known	Case II: σ unknown
$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$	$E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$ $\to df = n-1$
TI: ZInterval inpt: $\boxed{\text{Stats}}$	TI: TInterval inpt: $\boxed{\text{Stats}}$

$E = \frac{-}{2}$ $\bar{x} = \frac{+}{2}$

Jan 30-5:17 PM

Given: $n=32$, $\bar{x}=78$, $\sigma=12$, C-level 90%

Find conf. interval for pop. mean μ.

Since σ is known

Use Z Interval

STAT

TESTS

Inpt: STATS

$\sigma=12$

whole * → $\bar{x}=78$

$n=32$

C-level: .9

Calculate

→ Round to whole *

$$\boxed{75<\mu<81}$$

$$E=\frac{81-75}{2}=3$$

$$\bar{x}=\frac{81+75}{2}=78$$

Jan 30-5:24 PM

Given: $n=15$, $\bar{x}=125.8$, $S=9.5$

C-level: 98%

Find Conf. interval for population mean μ.

σ is unknown ⇒ Case II

$$\boxed{119.4<\mu<132.2}$$

T Interval

inpt: Stats

$\bar{x}=125.8$

$S=9.5$

$n=15$

C-level: .98

Calculate

$df=n-1$

$=\boxed{14}$

$$E=\frac{132.2-119.4}{2}=\boxed{6.4}$$

1 dec. Place

$$\bar{x}=\frac{132.2+119.4}{2}=\boxed{125.8}$$

Jan 30-5:30 PM

I randomly selected 30 college students, their mean age was 32.5 Yrs.

$n=30$, $\bar{x}=32.5$

Find 99% Conf. interval for the mean age of all students assuming the standard deviation of ages of all students is 12.5 Yrs.

C-level: .99

$< \mu <$

$\sigma = 12.5$

Since σ is known $\Rightarrow$ Z Interval

$\boxed{26.6 < \mu < 38.4}$

inpt: STATS

$\sigma = 12.5$

$\bar{x} = 32.5$ ← one-decimal

$n = 30$

C-level: .99

Calculate

$E = \dfrac{38.4 - 26.6}{2} = \boxed{5.9}$

$\bar{x} = \dfrac{38.4 + 26.6}{2} = \boxed{32.5}$

Jan 30-5:49 PM

I randomly selected 20 exams, their mean score was 88 with standard deviation 10.

$n=20$ $\bar{x}=88$ $S=10$

Find Conf. interval for mean score of all exams.

No C-level

$\Rightarrow$ Use .95

$< \mu <$

σ unknown $\Rightarrow$ T Interval

inpt: Stats

whole # → $\bar{x}=88$ → Round to whole #

$S=10$

$n=20$

C-level: .95

Calculate

$\boxed{83 < \mu < 93}$

$E = \dfrac{93 - 83}{2} = 5$

$\bar{x} = \dfrac{93 + 83}{2} = 88$

Jan 30-5:57 PM

I randomly selected 10 gas stations.

Here are the price of gas/gallon.

4.25	4.35	4.19
3.89	3.99	4.05
3.75	4.35	4.19
	3.85	

Sample

Store in L1

Find

$\bar{x} = 4.09$

$S = 0.21$

Find 99% Conf. interval for the mean gas price/ga. of all gas stations.

C-level : .99

σ unknown $\Rightarrow$ T Interval

inpt: STATS

$\bar{x} = 4.09$

$S = .21$

$n = 10$

C-level : .99

Calculate

$$\boxed{3.87 < \mu < 4.31}$$

$$E = \frac{4.31 - 3.87}{2} = \boxed{.22}$$

$$\bar{x} = \frac{4.31 + 3.87}{2} = \boxed{4.09}$$

Jan 30-6:05 PM

How to determine minimum sample size needed when working with pop. mean.

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

If we isolate n $\rightarrow$

$$\boxed{n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2}$$

If decimal, Always Round-Up

If σ is unknown, use S

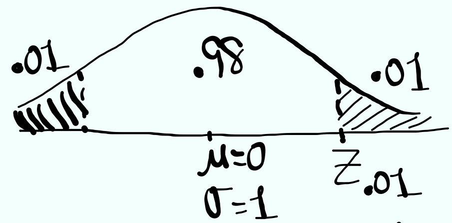
$$n = \left(\frac{z_{\alpha/2} \cdot S}{E}\right)^2$$

Jan 30-6:13 PM

Given C-level: 98%

$E = 8$

$\sigma = 15$



$z_{.01} = \text{invNorm}(.99, 0, 1)$

Find min. Sample Size needed to Construct Conf. Int. for pop. mean.

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2 = \left(\frac{2.326 \cdot 15}{8}\right)^2$$

$$= 19.021\ldots$$

$$\boxed{n = 20}$$

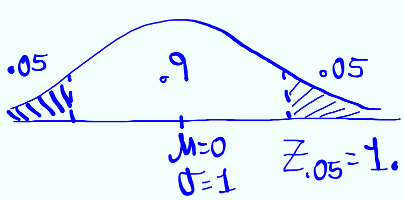
Jan 30-6:17 PM

Given C-level: 90% $E = 5$ $S = 18$

Find min. Sample Size needed to construct Conf. int. for pop. mean.

σ unknown $\rightarrow$ $n = \left(\frac{z_{\alpha/2} \cdot S}{E}\right)^2 = \left(\frac{1.645 \cdot 18}{5}\right)^2$

$= 35.07\ldots$



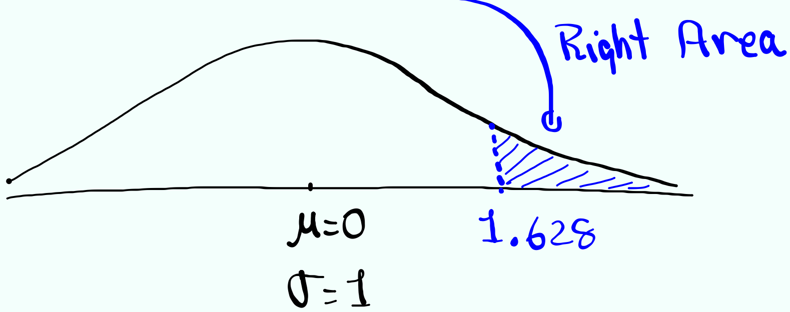
$z_{.05} = 1.645$

$\boxed{n = 36}$

SG 21 & SG 22

Jan 30-6:22 PM

$P(Z > 1.628)$

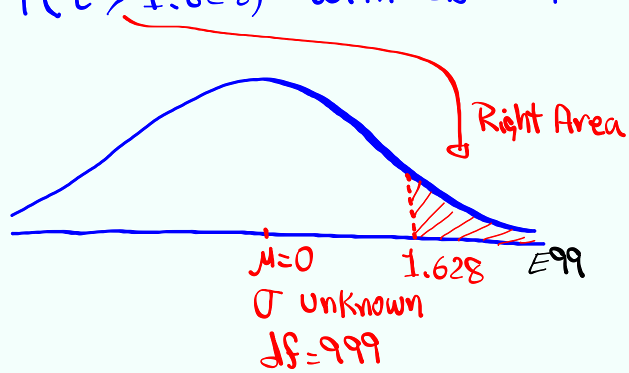


$= \text{normalcdf}(1.628, E99, 0, 1)$

$= \boxed{.052}$

Jan 30-6:29 PM

find $P(t > 1.628)$ with $df = 999$



$\text{tcdf}(\underset{\text{Lower}}{1.628}, \underset{\text{upper}}{E99}, \underset{df}{999})$

$\approx \boxed{.052}$

As df gets bigger

$t_{\alpha/2} \approx z_{\alpha/2}$

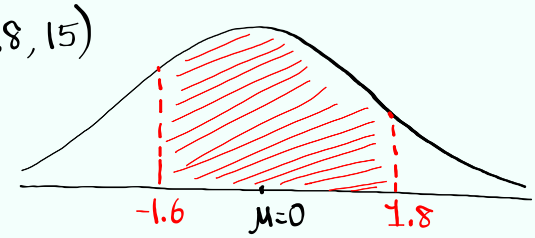
$t \approx z$

Jan 30-6:31 PM

Find $P(-1.6 < t < 1.8)$ with $df = 15$.

$= tcdf(-1.6, 1.8, 15)$

$= \boxed{.889}$

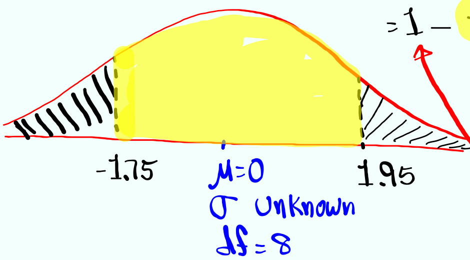


Find $P(t < -1.75 \text{ OR } t > 1.95)$ with $df = 8$.

$= 1 - tcdf(-1.75, 1.95, 8)$

Total Area

$= \boxed{.103}$



Jan 30-6:35 PM

χ^2 - Dist.

chi

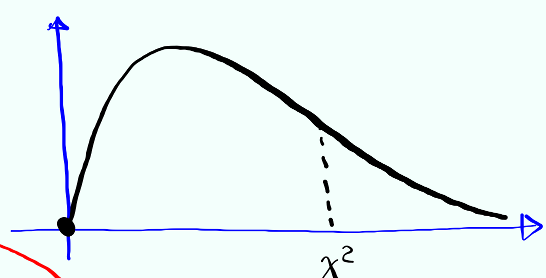
1) Graph begins at 0, and it is skewed to the right

2) Not symmetric, but total area 1.

3) It comes with degrees of freedom.

2nd

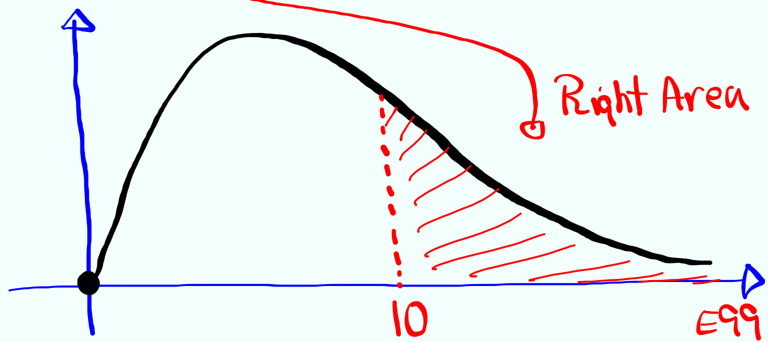
VARS



we use $\chi^2cdf(L, U, df)$

Jan 30-6:43 PM

find $P(\chi^2 > 10)$ with $df = 7$.

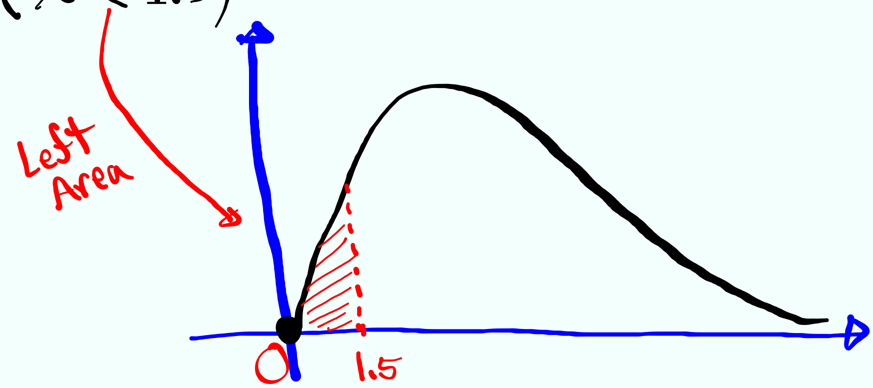


$$\chi^2cdf(\underset{L}{10}, \underset{U}{E99}, \underset{df}{7})$$

$$= \boxed{.189}$$

Jan 30-6:47 PM

find $P(\chi^2 < 1.5)$ with $df = 6$.



$$\chi^2cdf(0, 1.5, 6) = \boxed{.041}$$

Jan 30-6:50 PM

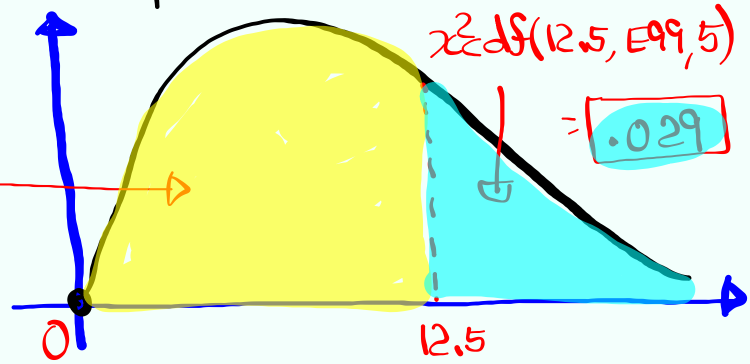
Find the area on each side of $\chi^2 = 12.5$ with $df = 5$, Multiply the Smaller area by 2.

$\chi^2cdf(0, 12.5, 5)$

$= \boxed{.971}$

$\chi^2cdf(12.5, E99, 5)$

$= \boxed{.029}$



$2 \cdot (\text{Smaller area}) = 2(.029) = \boxed{.058}$

Jan 30-6:52 PM